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Voltage unbalance numerical evaluation and minimization

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ABSTRACT

Among a series of parameters, power quality studies are concerned with voltage unbalances, which represent the voltage magnitude and phase deviation from nominal values. In order to determine the influence of the network's parameters on voltage unbalances, and to provide exact solutions to reduce or even eliminate them, the present study develops and presents two methods. First, a sensitivity analysis is used to determine the influence of each parameter, and then analytical solutions are developed in order to provide the changes needed for correction. The quantification index considered is the symmetrical components method.

1. Introduction

Over the years, electric power quality has become one of the major areas of research in electrical engineering. Of a series of power quality parameters, voltage unbalance has been considered as one of the main topics of study. This attention is especially due to the harmful effects this phenomenon has on tri-phase induction machines, which constitute the majority of industrial loads.

Several studies have been conducted in an attempt to understand the true consequences of the phenomenon. [1] presents the different quantification indexes created so far, and recommends the symmetrical components method as the most precise. [2] analyzes the effect of the sequences' angles on tri-phase induction machines. [3] and [4] show studies similar to [2], but take into consideration the magnitude of the positive sequence. [5] demonstrates the advantages of the NEMA quantification method, and [6] and [7] point out the characteristics and inconveniences of these quantification indexes.

These studies attempt to establish relationships between measuring methods and physical effects of the voltage unbalance. On the other hand, little effort has been made to determine the system's parameters influence on the phenomenon, and neither to calculate the variations needed in order to reduce the unbalance.

This paper presents and develops two methods. The first consists of the use of sensitivity analysis to determine the influence on the voltage unbalance for each parameter of the system (magnitudes and angles of the three phases), and the second changes the unbalance based on analytical solutions. The index used in both cases is the symmetrical components method.

The paper is organized as follows: in Section 2, the two methods are outlined; in Section 3, unbalance situations are analyzed in order to confirm the validity of these methods; and in Section 4, general conclusions are made.

2. Analysis

To date, four methods for quantifying voltage unbalances have been developed: NEMA, IEEE, symmetrical components and CIGRÉ. The first two were created based on the fact that several commercially available electricity meters were not able to measure the angular differences between each phase. The symmetrical components method relies on the Fortescue theorem, which represents an unbalanced tri-phase system through the sum of three balanced ones, and requires knowing the magnitudes and angular differences of all phases. The CIGRÉ method offers the same result as the symmetrical components method, but only needs the magnitudes of the voltages between phases.

The symmetrical components method is considered the most rigorous of the four, best reflecting the configuration of the system [1]. It is defined as the ratio between the magnitudes of the negative- and positive-sequence voltage components (V_2 and V_1, respectively). Eqs. (1)–(3) present the formulas for V_1, V_2, and for

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the symmetrical components method (K).

$$V_1 = |\tfrac{1}{3}(\overline{V_A} + a\overline{V_B} + a^2\overline{V_C})| \quad (1)$$

$$V_2 = |\tfrac{1}{3}(\overline{V_A} + a^2\overline{V_B} + a\overline{V_C})| \quad (2)$$

$$K = \frac{V_2}{V_1} \quad (3)$$

In Eqs. (1) and (2), $\overline{V_A}$, $\overline{V_B}$ and $\overline{V_C}$ are the phasors that represent the voltages in phases A, B and C, and $a = 1\angle 120°$.

Taking the above equations as a starting point, several developments can be made, simplifying the deduction of the methods presented later in this study. First, the phasors for phases A, B and C in Eqs. (1) and (2) are represented by their magnitudes (V_A, V_B and V_C, respectively) and angles (θ_A, θ_B and θ_C, respectively), as shown in Eqs. (4) and (5).

$$V_1 = \left|\frac{V_A\angle\theta_A + V_B\angle(\theta_B + 120°) + V_C\angle(\theta_C - 120°)}{3}\right| \quad (4)$$

$$V_2 = \left|\frac{V_A\angle\theta_A + V_B\angle(\theta_B - 120°) + V_C\angle(\theta_C + 120°)}{3}\right| \quad (5)$$

Next, after calculating the magnitudes indicated above, the square root and the multiplying term 1/3 can be eliminated, resulting in Eqs. (6) and (7).

$$9V_1^2 = \left\{V_A\cos\,\theta_A + V_B\cos(\theta_B + 120°) + V_C\cos(\theta_C - 120°)\right\}^2 + \left\{V_A\sin\,\theta_A + V_B\sin(\theta_B + 120°) + V_C\sin(\theta_C - 120°)\right\}^2 \quad (6)$$

$$9V_2^2 = \left\{V_A\cos\,\theta_A + V_B\cos(\theta_B - 120°) + V_C\cos(\theta_C + 120°)\right\}^2 + \left\{V_A\sin\theta_A + V_B\sin(\theta_B - 120°) + V_C\sin(\theta_C + 120°)\right\}^2 \quad (7)$$

The squared terms on the right side of Eqs. (6) and (7) can be further developed, resulting in Eqs. (8) and (9), where $\theta_{AB} = \theta_A - \theta_B$, $\theta_{BC} = \theta_B - \theta_C$ and $\theta_{CA} = \theta_C - \theta_A$.

$$9V_1^2 = V_A^2 + V_B^2 + V_C^2 + 2V_AV_B\cos(\theta_{AB} - 120°) + 2V_BV_C\cos(\theta_{BC} - 120°) + 2V_AV_C\cos(\theta_{CA} - 120°) \quad (8)$$

$$9V_2^2 = V_A^2 + V_B^2 + V_C^2 + 2V_AV_B\cos(\theta_{AB} + 120°) + 2V_BV_C\cos(\theta_{BC} + 120°) + 2V_AV_C\cos(\theta_{CA} + 120°) \quad (9)$$

Now, the right side of Eq. (3) can be multiplied and divided by three, squared and root-squared, resulting in Eq. (10).

$$K = \sqrt{\frac{9V_2^2}{9V_1^2}} \quad (10)$$

Eqs. (8)–(10) will be used throughout the rest of this work. In the next two sections, an analysis of the voltage unbalance and minimization methods will be presented.

2.1. Sensitivity of the voltage unbalance

Sensitivity analysis is a mathematical tool commonly used in electrical engineering, especially in control theory [8]. Its goal is to determine the change in a system subject to variations in its parameters. Mathematically speaking, it can be defined as a relationship between a parameter vector $\boldsymbol{\alpha} = (\alpha_1\alpha_2 \ldots \alpha_r)^T$ and a vector $\boldsymbol{\zeta} = (\zeta_1\zeta_2 \ldots \zeta_n)^T$, representing the dynamic response of the system. Furthermore, vectors $\boldsymbol{\alpha}$ and $\boldsymbol{\zeta}$ can be defined as the sum of two vectors, one being the nominal values (α_0 and ζ_0, respectively) and the other being the variations around the nominal values ($\Delta\alpha$ and $\Delta\zeta$, respectively). Eqs. (11) and (12) present the corresponding formulas.

$$\alpha = \alpha_0 + \Delta\alpha \quad (11)$$

$$\zeta = \zeta_0 + \Delta\zeta \quad (12)$$

The sensitivity equation (S) is defined as a mathematical relationship between the parameter variation and system response variation vectors, $\Delta\alpha$ and $\Delta\zeta$, around their nominal values, Eq. (13). This is a first-degree approximation, being valid under certain continuity conditions and for small parameter variations ($||\alpha_0|| \ll ||\Delta\alpha||$).

$$\Delta\zeta \approx S(\alpha_0)\Delta\alpha \quad (13)$$

The absolute and relative sensitivity functions, $S_{\alpha_j}^{\zeta_i}$ and $\bar{S}_{\alpha_j}^{\zeta_i}$, are presented in Eqs. (14) and (15), where $i = 1, 2, \ldots, n$, and $j = 1, 2, \ldots, r$.

$$S_{\alpha_j}^{\zeta_i} = \frac{\partial\zeta_i}{\partial\alpha_j}| \quad (14)$$

$$\bar{S}_{\alpha_j}^{\zeta_i} = \frac{\partial\zeta_i/\zeta_i}{\partial\alpha_i/\alpha_i}| = S_{\alpha_j}^{\zeta_i}\frac{\alpha_{j0}}{\zeta_{i0}} \quad (15)$$

Sensitivity analysis can be used to determine the influence of each phase parameter on the voltage unbalance. Defining α_0 as the magnitude and angle values of the three phases in an unbalanced situation, and ζ_0 as the corresponding K index for these values, the absolute and relative sensitivity functions can be obtained. They represent the change rate of the voltage unbalance, according to changes in each parameter, as indicated in Eqs. (14) and (15). A comparison of these rates indicates to which parameter the voltage unbalance is more sensitive. To avoid comparing terms having different dimensions, only the relative sensitivity function will be considered.

A direct application of Eq. (15) in (10) is presented in (16), where α_j can be equal to $V_A, V_B, V_C, \theta_A, \theta_B$ or θ_C. This produces very complex derivatives, requiring a few simplifications. First, the derivative of the square root is developed in (17). Next, the quotient rule for the derivative is applied, and the resulting expression is simplified in (18). A comparison between this last equation and (15) shows the presence of the expressions for the relative sensitivity functions of $9V_2^2$ and $9V_1^2$, as indicated in (19).

$$\bar{S}_{\alpha_j}^{K} = \alpha_j\sqrt{\frac{9V_1^2}{9V_2^2}}\frac{\partial}{\partial\alpha_j}\left(\sqrt{\frac{9V_2^2}{9V_1^2}}\right) \quad (16)$$

$$\bar{S}_{\alpha_j}^{K} = \frac{\alpha_j}{2}\frac{9V_1^2}{9V_2^2}\frac{\partial}{\partial\alpha_j}\left(\frac{9V_2^2}{9V_1^2}\right) \quad (17)$$

$$\bar{S}_{\alpha_j}^{K} = \frac{1}{2}\left(\frac{\alpha_j}{9V_2^2}\frac{\partial(9V_2^2)}{\partial\alpha_j} - \frac{\alpha_j}{9V_2^2}\frac{\partial(9V_1^2)}{\partial\alpha_j}\right) \quad (18)$$

$$\bar{S}_{\alpha_j}^{K} = \frac{1}{2}(\bar{S}_{\alpha_j}^{9V_2{}^2} - \bar{S}_{\alpha_j}^{9V_1{}^2}) \quad (19)$$

Eq. (19) shows that the relative sensitivity functions of K ($\bar{S}_{V_A}^{K}$, $\bar{S}_{V_B}^{K}$, $\bar{S}_{V_C}^{K}$, $\bar{S}_{\theta_A}^{K}$, $\bar{S}_{\theta_B}^{K}$ and $\bar{S}_{\theta_C}^{K}$, respectively, since α_j can be equal to V_A, V_B, V_C, θ_A, θ_B or θ_C) can be developed through the differentiation of (8) and (9), which turn out to be simpler expressions. For the sake of concision, the corresponding formulas will not be presented.

2.2. Minimization of the voltage unbalance

The sensitivity function indicates the influence of each system parameter on the corresponding voltage unbalance. It cannot be directly used to determine the parameter variation needed to reduce the voltage unbalance to a desired value because it is a first-degree estimate of a non-linear equation, K. Instead, analytical solutions are provided. Three situations are possible: variation of one parameter separately (magnitudes and angles of each phase), variation of two magnitudes simultaneously and variation of three magnitudes simultaneously.

For the first situation, Eq. (3) must be manipulated in order to isolate one of the variables. By squaring (10), bringing the term $9{V_1}^2$ to the left side of the equation, and substituting (8) and (9) in the corresponding expression, Eq. (20) is obtained.

$$\begin{aligned} K^2\{&V_A^2+V_B^2+V_C^2+2V_AV_B\cos(\theta_{AB}-120^\circ)\\ &+2V_BV_C\cos(\theta_{BC}-120^\circ)+2V_AV_C\cos(\theta_{CA}-120^\circ)\}\\ =\ &V_A^2+V_B^2+V_C^2+2V_AV_B\cos(\theta_{AB}+120^\circ)\\ &+2V_BV_C\cos(\theta_{BC}+120^\circ)+2V_AV_C\cos(\theta_{CA}+120^\circ) \end{aligned} \tag{20}$$

To hold this equality, one of the parameters, V_A, V_B, V_C, θ_A, θ_B or θ_C can be changed, providing a new value for K, here called K_{new}. For instance, isolating V_A in (20) results in a quadratic polynomial, (21). The terms A_{VA}, B_{VA} and C_{VA} are defined in Eqs. (22)–(24).

$$A_{VA}V_A^2+B_{VA}V_A+C_{VA}=0 \tag{21}$$

$$A_{VA}=1-K_{new}^2 \tag{22}$$

$$\begin{aligned} B_{VA}=\ &2V_B[\cos(\theta_{AB}+120^\circ)-K_{new}^2\cos(\theta_{AB}-120^\circ)]\\ &+2V_C[\cos(\theta_{CA}+120^\circ)-K_{new}^2\cos(\theta_{CA}-120^\circ)] \end{aligned} \tag{23}$$

$$\begin{aligned} C_{VA}=\ &2V_BV_C[\cos(\theta_{BC}+120^\circ)-K_{new}^2\cos(\theta_{BC}-120^\circ)]\\ &+(1-K_{new}^2)(V_B^2+V_C^2) \end{aligned} \tag{24}$$

The above equations show that, given initial values for V_B, V_C, θ_A, θ_B and θ_C, a different K value can be chosen (K_{new}), and two new values of V_A will satisfy this new voltage unbalance situation. These two solutions only have a physical meaning for real, positive values, such that $B_{VA}^2-4A_{VA}C_{VA}\geq 0$. In developing this equation and isolating K_{new}, limit values can be determined for the correction of the voltage unbalance. Eqs. (25)–(32) present the corresponding formulas.

$$A_{VAK}K_{new}^4+B_{VAK}K_{new}^2+C_{VAK}=0 \tag{25}$$

$$A_{VAK}=\alpha_{VAK}^2+4\gamma_{VAK} \tag{26}$$

$$B_{VAK}=2\alpha_{VAK}\beta_{VAK}+4(\delta_{VAK}-\gamma_{VAK}) \tag{27}$$

$$C_{VAK}=\beta_{VAK}^2-4\delta_{VAK} \tag{28}$$

$$\alpha_{VAK}=-2V_B\cos(\theta_{AB}-120^\circ)-2V_C\cos(\theta_{CA}-120^\circ) \tag{29}$$

$$\beta_{VAK}=2V_B\cos(\theta_{AB}+120^\circ)+2V_C\cos(\theta_{CA}+120^\circ) \tag{30}$$

$$\gamma_{VAK}=-2V_BV_C\cos(\theta_{BC}-120^\circ)-V_B^2-V_C^2 \tag{31}$$

$$\delta_{VAK}=2V_BV_C\cos(\theta_{BC}+120^\circ)+V_B^2+V_C^2 \tag{32}$$

Consequently, α_{VAK}, β_{VAK}, γ_{VAK} and δ_{VAK} are the parameters that indicate, by satisfying Eq. (25) and $k_{new}\geq 0$, the minimum k_{new} value that can be reached by changing V_A, given initial values for V_B, V_C, θ_A, θ_B and θ_C.

The same procedure can be repeated for the magnitudes of phases B and C, resulting in similar equations. As for the phase angles, the principle is the same, but the resulting equations are different. The corresponding equations for θ_B are presented in Eqs. (33)–(46). Eq. (33) indicates the new value of θ_B ($\theta_{B_{new}}$) that satisfies the chosen K_{new} value, given initial values for V_A, V_B, V_C, θ_A and θ_C. Eqs. (34)–(36) define the parameters for (33). Eq. (37) determines the minimum value of K_{new} that can be reached through $\theta_{B_{new}}$, and Eqs. (38)–(46) define parameters for (37).

$$\theta_{B_{new}}=\text{arc tg}\left(\frac{B_{\theta_B}}{A_{\theta_B}}\right)\pm\text{arc}\cos\left(\frac{-C_{\theta_B}}{\sqrt{A_{\theta_B}^2+B_{\theta_B}^2}}\right) \tag{33}$$

$$\begin{aligned} A_{\theta_B}=\ &2V_AV_B[\cos(\theta_A+120^\circ)-K_{new}^2\cos(\theta_A-120^\circ)]\\ &+2V_BV_C[\cos(\theta_C-120^\circ)-K_{new}^2\cos(\theta_C+120^\circ)] \end{aligned} \tag{34}$$

$$\begin{aligned} B_{\theta_B}=\ &2V_AV_B[\sin(\theta_A+120^\circ)-K_{new}^2\sin(\theta_A-120^\circ)]\\ &+2V_BV_C[\sin(\theta_C-120^\circ)-K_{new}^2\sin(\theta_C+120^\circ)] \end{aligned} \tag{35}$$

$$\begin{aligned} C_{\theta_B}=\ &(1-K_{new}^2)(V_A^2+V_B^2+V_C^2)+2V_AV_C[\cos(\theta_C-\theta_A+120^\circ)\\ &-K_{new}^2\cos(\theta_C-\theta_A-120^\circ)] \end{aligned} \tag{36}$$

$$A_{\theta_{BK}}K_{new}^4+B_{\theta_{BK}}K_{new}^2+C_{new}=0 \tag{37}$$

$$A_{\theta_{BK}}=\alpha_{\theta_{BK}}^2-\gamma_{\theta_{BK}}^2-\varepsilon_{\theta_{BK}}^2 \tag{38}$$

$$B_{\theta_{BK}}=2\alpha_{\theta_{BK}}\beta_{\theta_{BK}}-2\gamma_{\theta_{BK}}\delta_{\theta_{BK}}-2\varepsilon_{\theta_{BK}}\phi_{\theta_{BK}} \tag{39}$$

$$C_{\theta_{BK}}=\beta_{\theta_{BK}}^2-\delta_{\theta_{BK}}^2-\phi_{\theta_{BK}}^2 \tag{40}$$

$$\alpha_{\theta_{BK}}=-(V_A^2+V_B^2+V_C^2)-2V_AV_C\cos(\theta_C-\theta_A-120^\circ) \tag{41}$$

$$\beta_{\theta_{BK}}=V_A^2+V_B^2+V_C^2+2V_AV_C\cos(\theta_C-\theta_A+120^\circ) \tag{42}$$

$$\gamma_{\theta_{BK}}=-2V_AV_B\cos(\theta_A-120^\circ)-2V_BV_C\cos(\theta_C-120^\circ) \tag{43}$$

$$\delta_{\theta_{BK}}=2V_AV_B\cos(\theta_A+120^\circ)+2V_BV_C\cos(\theta_C+120^\circ) \tag{44}$$

$$\varepsilon_{\theta_{BK}}=-2V_AV_B\sin(\theta_A-120^\circ)-2V_BV_C\sin(\theta_C-120^\circ) \tag{45}$$

$$\phi_{\theta_{BK}}=2V_AV_B\sin(\theta_A+120^\circ)+2V_BV_C\sin(\theta_C+120^\circ) \tag{46}$$

The solutions for variables V_B, V_C and θ_C will not be developed, for the sake of concision, and θ_A is not considered, since it is used by electricity meters as the reference angle, being always null.

Next, the method for correcting the voltage unbalance through three magnitudes is developed. There are now three variables and only one equation, (20), yielding infinite solutions. To solve this problem, a different approach is needed. The goal now is to find a solution that results in the smallest variation for the three magnitudes. Considering the Euclidean distance, this minimization problem is stated as: find $\min((V_A-V_{A_0})^2+(V_B-V_{B_0})^2+(V_C-V_{C_0})^2)$, subject to Eq. (20), where V_{A_0}, V_{B_0} and V_{C_0} are the initial values of the magnitudes of phases A, B and C.

Using Laplace's method, a new variable (λ) is introduced in order to include (20), creating a new function to be minimized, Eq. (47).

$$\begin{aligned} \min\{&(V_A-V_{A_0})^2+(V_B-V_{B_0})^2+(V_C-V_{C_0})^2\\ &+\lambda\{(V_A^2+V_B^2+V_C^2)(1-K_{new}^2)\\ &+2V_AV_B[\cos(\theta_{AB}-120^\circ)-K_{new}^2\cos(\theta_{AB}+120^\circ)]\\ &+2V_BV_C[\cos(\theta_{BC}-120^\circ)-K_{new}^2\cos(\theta_{BC}+120^\circ)]\\ &+2V_AV_C[\cos(\theta_{CA}-120^\circ)-K_{new}^2\cos(\theta_{CA}+120^\circ)]\}\} \end{aligned} \tag{47}$$

Calculating the derivatives of (47) in respect to V_A, V_B, V_C and λ, and making them equal to zero, a system with four equations

Table 1
Relative sensitivity of K and minimum values for correction through each parameter, for a system with one unbalanced magnitude.

	V_A	V_B	V_C	θ_B	θ_C
Relative sensitivity (dimensionless)	−10.89	5.45	5.45	−21.00	21.00
Minimum K (%)	0	2.605	2.605	1.383	1.383
Corresponding values for minimum K	220 V	211.164 V	211.164 V	−124,431°	124,431°

and four variables is created. The solutions are too cumbersome to be developed by hand, requiring a software that calculates operations with symbolic math, such as Maple®, MatLab® and SciLab®.

The procedure for obtaining the correction of a voltage unbalance through two-phase magnitudes is the same as above, but with one less variable. For instance, if the magnitude of phase C should remain constant, then the function to be minimized is $((V_A - V_{A_0})^2 + (V_B - V_{B_0})^2)$, subject to (20). The method for choosing the constant magnitude phase was the one closest to the nominal voltage value.

3. Results

In order to verify the validity of the methods developed in the previous section, different sets of unbalanced voltages were studied. Three situations were chosen: one unbalance magnitude; one unbalance angle; three unbalanced magnitudes and two unbalanced angles. In all of these, the angle of phase A was taken as a reference for the other two phasors, being always equal to zero, and the nominal magnitude considered was 220 V, with no loss of generality.

3.1. One unbalanced magnitude

The first situation considered for validating the methods was a system with all parameters balanced, except for the magnitude of phase A. Eq. (48) presents the chosen values.

$$\begin{aligned} \overline{V_A} &= 201\angle 0^\circ \\ \overline{V_B} &= 220\angle -120^\circ \\ \overline{V_C} &= 220\angle 120^\circ \end{aligned} \tag{48}$$

According to Eqs. (1)–(3), this system results in a voltage unbalance of 2.96%. The relative sensitivities (presented in Section 2.1), the minimum values for correction through one parameter and the corresponding parameter values that result in these minimum K values (Section 2.2) are presented in Table 1.

Table 1 indicates positive and negative values for the relative sensitivities. The negative sign is determined by the growth direction of the derivative of K, according to each parameter. For instance, increasing V_A leads to a positive percentage increase for this variable ($\Delta V_A > 0$), and according to the relative sensitivity value, to a negative percentage increase in K ($\Delta K < 0$). In other words, an increase in V_A reduces K.

Ignoring the negative signs, it can be seen that, for this voltage unbalance situation, K is much more sensitive to the angles than to magnitudes of the phases, even though the only unbalanced parameter is V_A. Among the phase magnitudes, K is almost twice as sensitive to V_A as to V_B and V_C. Despite the higher sensitivity to angles, the correction through these parameters is limited to 1.383%, while K can be made null through V_A, which is an expected result.

A series of K values were chosen, in order to validate the results in Table 1. The relative sensitivities were approximated by $(\Delta K/K)/(\Delta\alpha_i/\alpha_i)$, and the correction values were determined by the methods presented in Section 2.2. The parameter values that result in the minimum K values are V_A = 220 V, V_B = 211.164 V, V_C = 211.164 V, θ_B = −124.431° and θ_C = 124.431°. Each of these values should be considered separately, keeping all the others constant, at their initial values. The sensitivity analysis and the methods of correction through one variable were confirmed. As for the methods of correction through two and three magnitudes, correction was possible for all K values, including for K = 0%. The magnitude values for K = 0% were V_A = 220 V, V_B = 220 V, V_C = 220 V, for the correction through two magnitudes, and V_A = 213.667 V, V_B = 213.667 V, V_C = 213.667 V, for the correction through three magnitudes. In the first case, V_B was kept constant, because it was closer to the nominal value, and the second case is the correction that generates the smaller Euclidean distance through the three magnitudes.

3.2. One unbalanced angle

The second situation considered was a system with the angle of phase C unbalanced (Eq. (49)). It resulted in a 2.328% voltage unbalance. Table 2 presents the relative sensitivities, the minimum values for correction through one parameter, and the corresponding values for these parameters.

$$\begin{aligned} \overline{V_A} &= 220\angle 0^\circ \\ \overline{V_B} &= 220\angle -120^\circ \\ \overline{V_C} &= 220\angle 116^\circ \end{aligned} \tag{49}$$

Table 2 indicates that the voltage unbalance is highly sensitive to θ_C, but practically insensitive to V_C . Furthermore, it shows that it is impossible to reduce the voltage unbalance through V_C, but possible to eliminate the unbalance through θ_C, as expected. Once again, K is more sensitive to angles than to magnitudes.

As in Section 3.1, the limits and sensitivity values in Table 2 were validated for a series of K values, using the same procedures. As for the methods of correction through two and three magnitudes, correction was again possible for all K values, including for K = 0%. This was an unexpected result, which reveals that when the angles are kept unbalanced, it is possible to find a set of magnitudes that make the voltage unbalance null, according to Eq. (3). The magnitude values for K = 0% were V_A = 220 V, V_B = 202.926 V, V_C = 211.979 V, for the correction through two magnitudes, and V_A = 228.447 V, V_B = 210.718 V, V_C = 220.119 V, for the correction through three magnitudes. Clearly, these voltage phasors are unbalanced, even though they yield K = 0%.

Further analysis of the latter results indicates that they represent tri-phase systems with V_2 = 0 V, but $V_0 \neq$ 0 V. Additionally, the cor-

Table 2
Relative sensitivity of K and minimum values for correction through each parameter, for a system with one unbalanced angle.

	V_A	V_B	V_C	θ_B	θ_C
Relative sensitivity (dimensionless)	−12.48	12.31	0.17	−14.10	29.02
Minimum K (%)	1.210	1.116	2.328	2.036	0
Corresponding values for minimum K	232.920 V	206.678 V	220 V	−121.976°	120°

Table 3
Relative sensitivity of K and minimum values for correction through each parameter, for a system with three unbalanced magnitudes and two unbalanced angles.

	V_A	V_B	V_C	θ_B	θ_C
Relative sensitivity (dimensionless)	−11.16	−0.48	11.64	−28.78	16.05
Minimum K (%)	1.169	2.493	1.365	0.119	2.081
Corresponding values for minimum K	215.332 V	220 V	217.576 V	−126.218°	123.267°

responding line voltages of these systems are perfectly balanced. So, it can be concluded that the methods of correction through two or three magnitudes are able to make V_2 null, eliminating the voltage unbalance (according to Eq. (3)), but they are not responsible for the final value of V_0.

3.3. Three unbalanced magnitudes and two unbalanced angles

The last situation considered was a system with all parameters unbalanced (Eq. (50)), which results in $K = 2.49\%$. Table 3 presents the relative sensitivities, the minimum values for correction through one parameter, and the corresponding values for these parameters.

$$\begin{aligned} \overline{V_A} &= 201\angle 0^\circ \\ \overline{V_B} &= 220\angle -122^\circ \\ \overline{V_C} &= 231\angle 121^\circ \end{aligned} \tag{50}$$

Table 3 indicates extremely low K sensitivity to V_B, medium sensitivity to V_A, V_C and θ_C, and high sensitivity to θ_B. Actually, the voltage unbalance remains unaltered by changing V_B, and can be made almost null by changing only θ_B. As in Sections 3.1 and 3.2, K remains more sensitive to voltage angles than magnitudes.

The limits and sensitivity values in Table 3 were validated, including for $K = 0\%$. No parameter altered alone was able to eliminate the voltage unbalance, but θ_B could lead K to 0.119%. As for the methods of correction through two and three magnitudes, correction was again possible for all K values, including for $K = 0\%$, yielding unbalanced systems with $V_2 = 0$ V, but $V_0 \neq 0$ V.

4. Conclusions

In this paper, mathematical models and algorithms have been proposed and developed in order to analyze and correct voltage unbalances, based on the symmetrical components method. Two methodologies were created: sensitivity analysis and analytical solutions. The first was used to determine the influence of each system parameter on the voltage unbalance, and the second provided exact solutions for the correction of the unbalance.

Both methods were applied to three different unbalanced situations, in order to be validated. The systems considered had the following unbalanced parameters: one magnitude, one angle, and three magnitudes and two angles (where phase A was always taken as the angular reference). Two special aspects of the K index were verified after the study of each unbalance situation. First, K presented higher sensitivity to voltage angles than to magnitudes. Second, whenever there was an angle unbalance, it was possible to find phasor sets that yielded $K = 0\%$, keeping the same angle unbalance. That is, there were unbalanced phasor sets that have a null K index, but whose line voltages are balanced. This is an undesired aspect of the symmetrical components method, since it is unable to detect the voltage unbalance of certain special situations.

The sensitivity analysis held true for all cases, indicating the influence of each parameter, but it did not indicate any limits for correction, whereas the analytical solutions provided this kind of information. The correction through two and three magnitudes indicated unusual aspects of the K index, as mentioned before. The combination of both methodologies constitutes a valuable tool for the analysis of voltage unbalances.

Appendix A. List of symbols

K	voltage unbalance factor (symmetrical components method)
$S_{\alpha_j}^{\zeta_i}$	absolute sensitivity function
$\bar{S}_{\alpha_j}^{\zeta_i}$	relative sensitivity function
V_1	magnitude of the positive-sequence voltage component
V_2	magnitude of the negative-sequence voltage component
$\overline{V_A}, \overline{V_B}, \overline{V_C}$	voltage phasors of phases A, B and C

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