Analysis of the Complex Voltage Unbalance Factor Behavior Resulting from the Variation of Voltage Magnitudes and Angles

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Abstract--**This paper presents the main aspects observed in analyzing Complex Voltage Unbalance Factor (CVUF) behavior resulting from the variation of voltage magnitudes and angles. The goal is to identify possible incoherencies regarding the use of the CVUF, and also to investigate whether this factor is more sensitive to variations in magnitude or angles under various voltage unbalance conditions. This study also evaluates the efficiency of the use of the CVUF angle and its association with positive component magnitudes. The results indicate that the CVUF should not be used as a single and sufficient parameter for the quantification of voltage unbalances, which highlights the need to develop a new indicator which may establish a more clear and simple association between this disturbance and its effects on electrical equipment.**

Index Terms--**Voltage Unbalance, Complex Voltage Unbalance Factor, Complex Voltage Unbalance Factor Behavior, Variations in Voltage Magnitudes and Angles.**

I. Nomenclature

$\overline{V_A}, \overline{V_B}, \overline{V_C}$ – line-to-neutral voltage phasors of phases A, B and C;

$\overline{V_0}, \overline{V_1}, \overline{V_2}$ – voltage phasors of the components of the zero, positive and negative sequences;

a – Fortescue operator, having unitary magnitude and angle equal to 120º;

VUF – voltage unbalance factor (symmetrical components method);

$CVUF$ – complex voltage unbalance factor (symmetrical components method).

II. Introduction

SEVERAL attempts have been made to identify the effects of voltage unbalances on electrical installation loads [1]-[3]. Voltage unbalances provoke, among other problems, excessive losses, overheating, insulation degradation, and a reduction in the lifespan of motors and transformers. Equipment protection systems are also affected, which result in interruptions in production processes. Therefore, any solution aimed at maximizing the lifespan and the working efficiency of motors and transformers in the event of electrical system disturbances should be supported by adequate knowledge and quantification of the parameters involved.

In this regard, the following aspects should be considered:

- Current methodologies neither are adequate nor present clear justifications regarding their choices and applications. Therefore, voltage unbalance quantification should be based on methods which assume a strong correlation with possible effects on electrical system equipment;
- The wide range of models and characteristics of modern equipment, which produce different sensitivity levels when submitted to voltage unbalances;
- The absence of performance or immunity standards for equipment submitted to different voltage unbalance conditions; and
- The high costs which may be involved with this phenomenon.

In this context, this study aimed: 1) to present the main results obtained from the analyses of the effects of voltage magnitude variations on CVUF behavior; 2) to identify the possible incoherencies regarding its use; 3) to investigate whether unbalances are more sensitive to variations in voltage magnitudes than to variations in angles under different unbalance conditions; and 4) to evaluate the efficiency of 2 methods suggested by [1] and [2], namely the use of the CVUF angle, and the association of the positive component magnitude with the CVUF.

Presented below are the equations of the unbalance quantification method used in this study and the structure used to perform the simulations involving the CVUF.

III. The Complex Voltage Unbalance Factor

In this study, the symmetrical component method will be used. It is allegedly the most reliable method of voltage unbalance quantification [4], due to the simultaneous utilization of voltage magnitudes and angles.

Let us consider V_a, V_b, and V_c as the set of line-to-neutral voltage phasors. The components of the zero, positive and negative sequences, respectively V_0, V_1, and V_2, may be obtained from the following transformation (equation 1):

$$\begin{bmatrix} V_0 \\ V_1 \\ V_2 \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} \qquad (1)$$

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where *a* is the Fortescue operator, having unitary magnitude and angle equal to 120º.

In the symmetrical component method, the voltage unbalance factor is defined as the relation between the magnitudes of the negative V_2 and positive V_1 sequences, according to equation (2).

$$VUF = \left| \frac{V_2}{V_1} \right| * 100\% \qquad (2)$$

Since the components of the negative and positive sequences are phasors, their ratio results in a complex number, the CVUF (equation 3), whose angle represents the lag between the negative and positive sequences.

$$CVUF = \frac{\bar{V}_2}{\bar{V}_1} * 100\% \qquad (3)$$

In the next section, the boundary conditions and the methodology used for the computational simulations are outlined.

IV. Analysis of the CVUF

In the simulations conducted in this study, the following voltage unbalance conditions were used: constant voltage angles, constant voltage magnitudes, and the variation of one voltage magnitude and one angle. Due to the large volume of information obtained, only those results related to the analysis of the voltage magnitude variations are presented. The most relevant aspects identified are shown at the end of this section.

It is important to emphasize those magnitudes of 220 volts, and phase angles A, B and C equal to 0º, -120º and +120º, respectively, were assumed as the nominal voltage values in our simulation, in accordance with Brazilian electrical system regulation.

A. Two-phase voltage magnitude variations

Figs.1 and 2 illustrate the behavior of the VUF resulting from the two-phase voltage magnitude variations, with the other parameters remaining constant. It may be seen that there are several voltage magnitude combinations that generate the same VUF. All points found on the level curves have the same unbalance values. Further analysis of Figs.1 and 2 indicates that voltage magnitude variations of approximately 10% in each phase generally produced VUF values of up to 6%. In fact, VUF values between 0 and 4% are determined only when the magnitude of one phase varies. However, these values are 0 and 6% when two phases undergo simultaneous variations of 10% in their magnitudes.

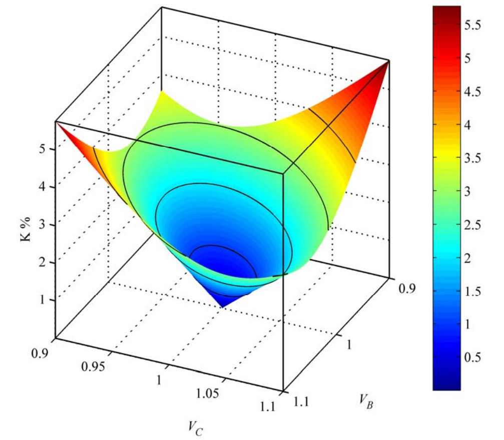


Fig. 1 - Behavior of the VUF considering variations in the voltage magnitudes of two phases

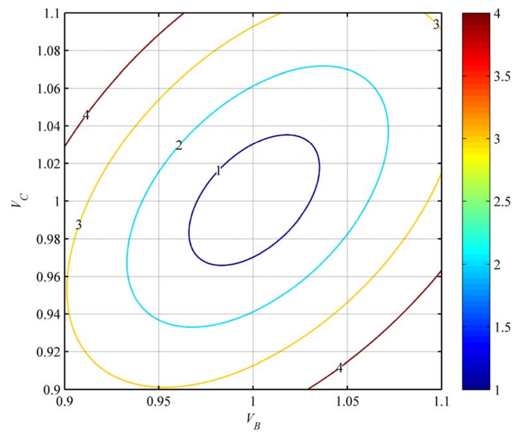
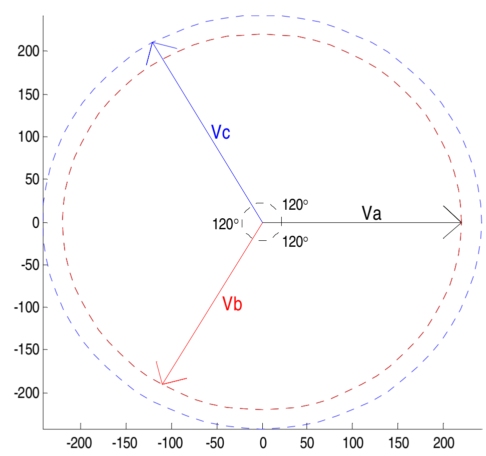
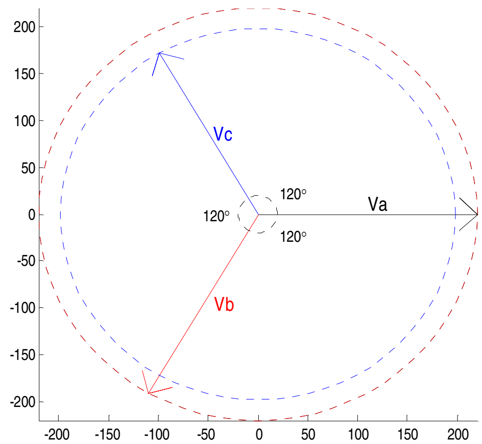


Fig. 2 – Behavior of the VUF considering variations in the voltage magnitudes of two phases – Level curves

Fig. 3 represents two unbalanced three-phase systems with the phase C magnitude 10% greater and 10% lower than the base value.



(a) - V_C=1.10*V_{BASE}



(b) - V_C=0.90*V_{BASE}
Fig. 3 – Unbalanced three-phase systems

The VUF values, which were calculated considering the unbalance conditions outlined in items (a) and (b) of Fig. 3 are, respectively, 3.226% and 3.448%. Thus, it may be concluded that an increase in the voltage magnitude of one or two phases generates a lower VUF than an equally proportional reduction in the magnitudes of the same phases.

Figs. 4 and 5 illustrate the CVUF angle behavior resulting from the two-phase voltage magnitude variations.

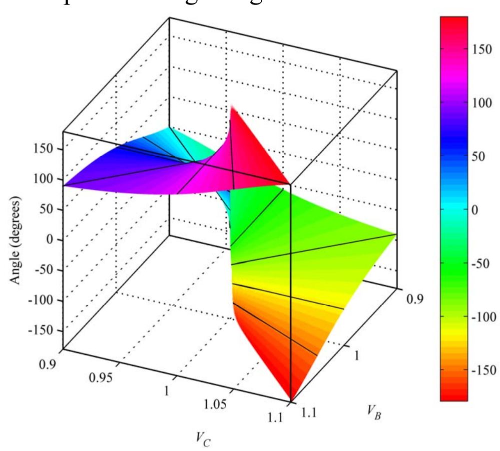


Fig. 4 – Behavior of the CVUF angle considering variations in the voltage magnitudes of two phases.

From the analysis of Figs.4 and 5 it is possible to conclude that there is one set of voltage phasors which have two-phase voltage magnitudes above the base value, making it impossible to define the exact value of the CVUF angle.

From Fig. 5 it can be verified that:

- Under unbalance conditions in which the two-phase voltage magnitudes are above the base value, the CVUF angles are determined in the first quadrant (going from 120° to 240°);
- Under unbalance conditions in which the magnitude of one phase is above the base value and the other is below, the CVUF angles are determined in the second and forth quadrants (between -120° and -60° and between 60° and 120°, respectively); and
- Under unbalance conditions in which the voltage magnitudes of two phases are below the base voltage, the CVUF angles are generated in the third quadrant (between -60° and 60°).

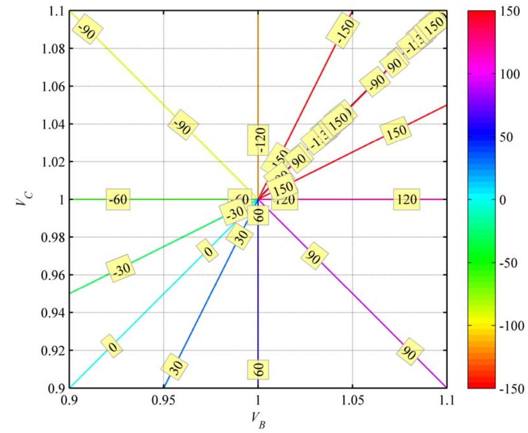


Fig. 5 - Behavior of the CVUF angle considering variations in the voltage magnitudes of two phases – Level curves

Therefore, it can be concluded that under unbalance conditions in which the nominal voltage angle values of each phase remain constant, the VUF can be used to quantify the unbalance in the system, with the CVUF angle indicating the condition, or conditions, generating the voltage unbalance.

B. Results of all simulations

Table I presents the VUF values for variations of 10% in the respective nominal one- and two-phase magnitudes and angles. These values indicate that the VUF is more sensitive to voltage angle variations than to variations in voltage magnitudes.

TABLE I
VUF VALUES FOR CERTAIN SIMULATED CONDITIONS

	Number of phases	VUF (%)
Magnitude	One	3.448
	Two	5.774
Angle	One	7.003
	Two	12.92

In all simulations, certain voltage combinations produced identical VUFs. Consequently, in the next part of the study, we shall attempt, based on the results obtained, to theoretically identify which of these unbalance conditions produced the same VUF values. In the graphs shown below, it is possible to visualize the geometric loci of the voltage magnitudes and angles for similar VUF magnitudes.

Evidently, there are three sets of conditions (variations in the angles, variations in the magnitudes, or simultaneous variations in both) under which a constant value may be attributed to the VUF - 2%, for example, which is the limit established by most standards. Our study established the following two simulation conditions:

- Voltage angles were kept constant with variations in the magnitudes; and
- Voltage magnitudes values were kept constant with variations in angles.

Each of the above voltage conditions produces a set of equal VUF values. It must be pointed out that simultaneous variations in magnitudes and angles are not considered in this study due to the difficulties in evaluating a system with five parameters varying at the same time. In addition, we present the results of the geometric loci of the voltage magnitudes, considering an angle variation of 1° around their nominal values. We also show the results of the geometric loci of the angles for conditions under which voltage magnitudes were kept constant at 201 and 231 volts.

C. Graphical analysis of magnitude variations with angles kept at their nominal values

Fig. 6 illustrates the geometric locus of the voltage magnitudes which produces VUF values of 2%, for the given voltage interval (201 to 231 volts). This interval was chosen because it represents the RMS voltage variation limits accepted for Brazilian electrical systems, for which the nominal voltage is 220 volts.

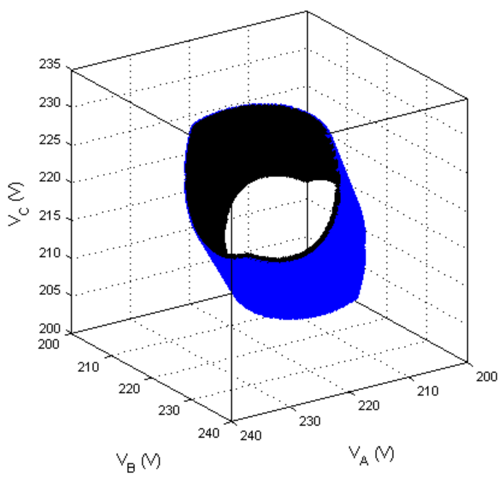


Fig. 6 – Geometric locus of voltage magnitudes which culminate in a VUF equal to 2%, considering constant angles.

It can be observed in Fig. 6 that the geometric loci of the voltage magnitude for a VUF equal to 2%, considering angles equal to 0º, -120º and 120º, generates a cylinder. The length of the cylinder shown in Fig. 6 depicts voltage magnitudes varying between 201 and 231 volts. When variations surpass the 201 and 231 volt limits, several other phasors then comprise the set that generates VUF=2%. Voltage magnitude values which are within and outside the cylinder generate unbalances where the VUF value is, respectively, less than and greater than 2%.

In order to simplify our analyses, the variation in voltage magnitude depicted by the curve in Fig. 6 is measured in p.u., with the base voltage value equal to the phase A value.

Fig. 7 shows the geometric locus of the voltage normalized by VA, for the constant and nominal angle values, and the VUF equal to 2%.

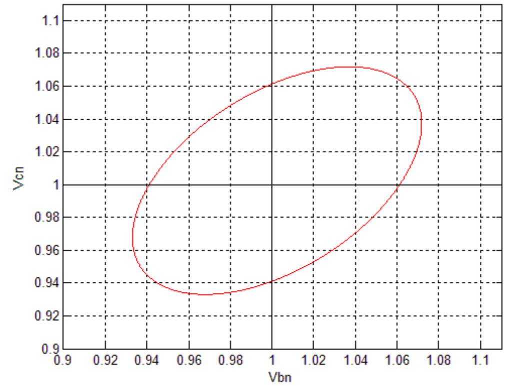


Fig. 7 – Geometric locus of voltages normalized by V_A which culminates in a VUF equal to 2%, angles held constant.

The corresponding geometric locus is an ellipse, centered at point (V_B/V_A, V_C/V_A) = (1,1). In fact, this is one of the level curves shown in Fig. 1, when the behavior of the VUF was analyzed considering the variations in voltage magnitudes. It must be pointed out that for every V_A value at a constant nominal voltage angle value, with the VUF equal to 2%, there are several voltage combinations which may produce different effects when applied to electrical power system equipment.

D. Graphical analysis of magnitude variations, 1° angle deviation

Fig. 8 illustrates the geometric loci of the voltage magnitudes for a VUF equal to 2%, considering situations in which the angles are kept constant, with variations up to 1º. The CVUF angle values for certain unbalance conditions may also be seen in the curves in Fig. 8

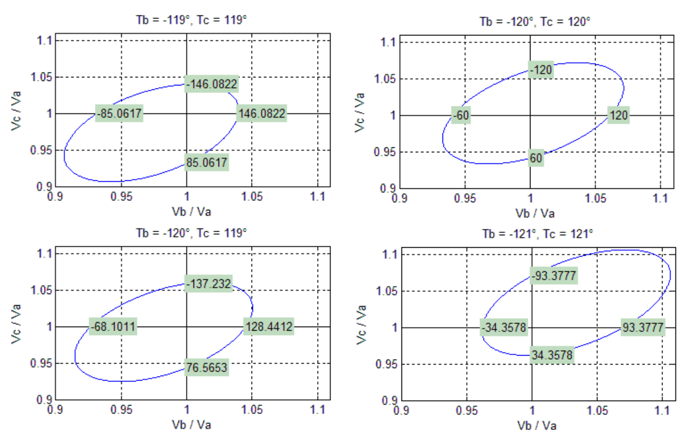


Fig. 8 – Geometric loci of magnitudes for certain voltage angle values

In Fig. 8 we can observe that, as the values of the voltage angles change, the ellipses formed by the magnitudes of phases A, B and C move in space. It can also be verified that the CVUF angles vary with changes in voltage angles. In fact, it can be proven that CVUF angles, which constitute the first quadrant for phase B and C voltages above the base value, are modified with changes in voltage angles. Therefore, it can be concluded, considering the different voltage angle values, that the CVUF angles, unless under the condition in which only the magnitudes vary, do not offer useful information regarding voltage unbalances.

E. Graphical analysis considering the variations of voltage angles, with magnitudes at their nominal values

Fig. 9 illustrates the geometric locus of the voltage angles for the VUF equal to 2%, with the voltage magnitude equal to 220 volts.

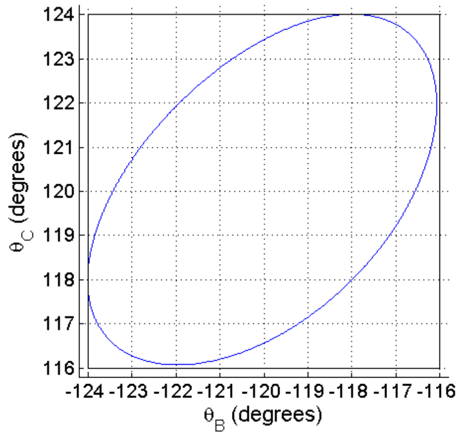


Fig. 9 – Geometric locus of voltage angles that culminate in a VUF equal to 2%, with magnitudes equal to 220 volts.

It can be observed from Fig. 9 that the geometric locus of the voltage angles for the VUF equal to 2%, considering the magnitudes equal to 220 volts, is an ellipse. In the angle analysis, the phase A angle was adopted as a reference (0^o).

It can be concluded that in spite of the fact that the nominal magnitude values remained constant, there are several angle values which will produce a voltage unbalance with a constant VUF.

F. Graphical analysis considering the variation of voltage angles and magnitudes at 201 and 231 volts

Figs. 10 and 11 illustrate the geometric loci of the voltage for a VUF equal to 2%, considering the condition in which the voltage magnitudes are kept constant at 201 and 231 volts, respectively.

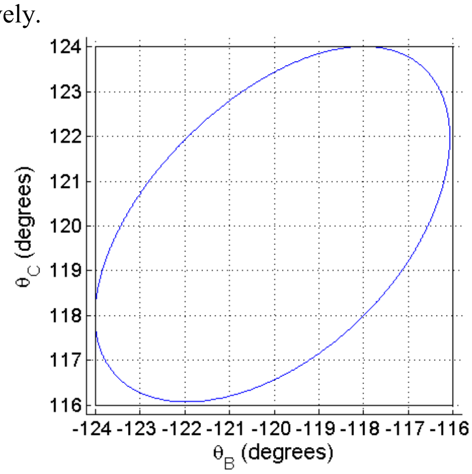


Fig. 10 – Geometric locus of voltage angles that culminate in a VUF equal to 2%, with magnitudes equal to 201 volts.

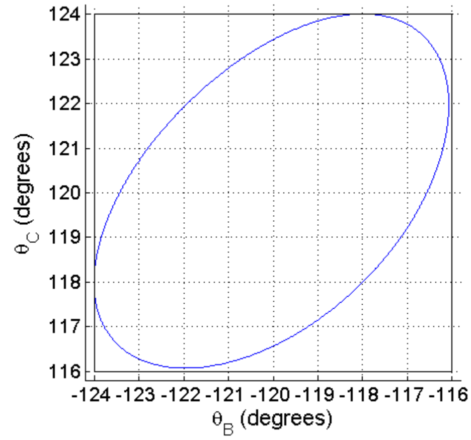


Fig. 11 – Geometric locus of voltage angles that culminate in a VUF equal to 2%, with magnitudes equal to 231 volts.

In Figs.9, 10 and 11 it can be verified that the geometric loci of the voltage angles, for a VUF equal to 2%, with magnitudes held constant and equal to 220, 201 and 231 volts, are identical. This was expected, considering that the VUF does not depend on the absolute voltage magnitude levels, but on the relative levels.

V. Critical Analysis of the Geometric Loci of Voltage Magnitudes and Angles for Equal VUFs

In this section, two phenomena observed in the simulations are shown, which support the questioning whether the VUF is an adequate index for the quantifying of voltage unbalances.

First, we consider the values shown in Table 2 that presents voltage magnitudes and angles which culminate in a VUF equal to 0%.

Table 2 – Voltage Phasors whose VUFs are equal 0%

Magnitudes (volts)			Angles (º)		
Phase A	Phase B	Phase C	Phase A	Phase B	Phase C
220	217.84	215.61	0	-119	120
220	224.48	222.27	0	-120	121
224.32	217.77	217.77	0	-119	119
222.19	220.01	217.76	0	-119	120

It may be stated from Table 2 that certain voltage magnitude and angle combinations may generate a VUF equal to 0%. It is probable that common loads, such as induction machines, may be adversely affected by these voltage values.

Secondly, based on the results obtained from the analyses made in the previous sections, we present another critical analysis of the phasors which, when applied to electrical equipment, will probably generate different effects.

Figs. 12 and 13 illustrate the geometric loci of the voltage magnitudes for a VUF equal to 2% and a VUF equal to 1.5%, respectively, with angles kept constant at their nominal values, and with a variation range of 201 to 231 volts (which are the RMS limit values accepted by Resolution 505 of ANEEL – the Brazilian National Electric Power Agency – for a nominal voltage of 220 volts). The points highlighted in the graphs constitute the voltages which present at least one module at 201 or 231 volts. The choice of the premises used in this analysis is justified for allowing the identifying, considering the respective 2% and 1.5% limits, of the voltage phasors which may produce different effects when applied to electrical power system equipment.

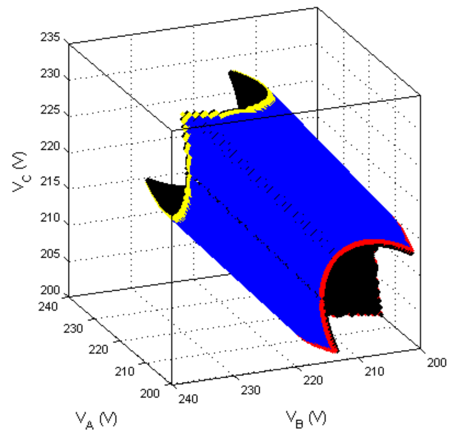


Fig. 12 – Geometric locus of the voltage magnitudes for a VUF equal to 2%, with emphasis on the voltages that present at least one level at 231 or 201 volts.

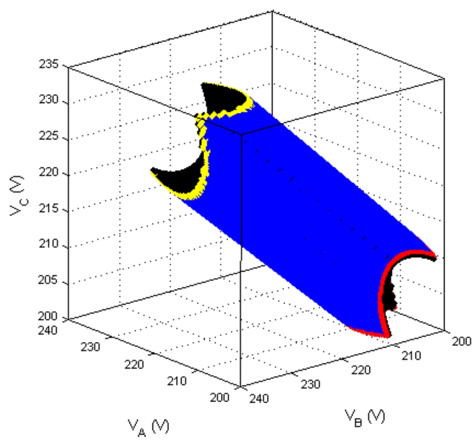


Fig. 13 - Geometric locus of the voltage magnitudes for a VUF equal to 1.5%, with emphasis on the voltages that present at least one level at 231 or 201 volts.

It can be observed, in Figs.12 and 13, that there are several voltage values for a VUF equal to 2% and a VUF equal to 1.5%, which present at least one magnitude at 201 or 231 volts. As previously mentioned, such curves are related to the conditions under which the angles are maintained constant at their nominal values.

It can be verified that the phasors which lead to a VUF equal to 2% and a VUF equal to 1.5%, respectively, may be within an extensive range of magnitude and angle variations.

From this set of samples, there may be phasors which, though generating a VUF equal to 1.5%, seem to be more harmful to loads such as induction machines, than in others which produce a VUF equal to 2%. This is the case of the phasorsV_A=201∟0º, V_B=201∟-21.32º and V_C=201∟+117.05º (for a VUF equal to 1.5%), compared to V_A=220∟0º, V_B=220∟-121.51º and V_C=220∟+116.10º (for a VUF equal to 2%).

These observations reinforce the need for further research aimed at detecting the effects of voltage unbalances on equipment, under different unbalance conditions. Such procedures may emphasize the fact that analyses which only consider the VUF may lead to incorrect assessments of the effects provoked by unbalances on electrical equipment. Furthermore, this kind of research increases the number of possibilities of reaching a solution for the problem in question.

VI. Analysis of the Use of V_1 Associated with the VUF

Once the geometric loci of some of the different phasors which generate the same VUF were identified, a group of researchers [3] proposed the use of the positive component magnitude V_1 together with the VUF for a more precise identification of the effects of unbalances on electrical system equipment.

Fig. 14 illustrates the geometrical locus of the voltage magnitudes for V_1 equal to 210, 215 and 220 volts, under conditions where the VUF is equal to 2%.

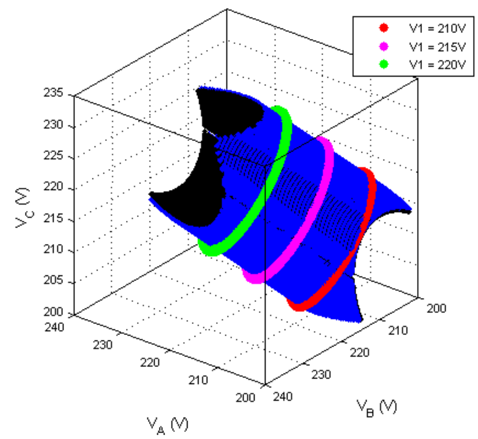


Fig. 14 – Geometric locus of the voltage magnitudes for a VUF equal to 2%, with emphasis on the voltages that present at least one level at 231, 215 or 201 volts

It can be seen in Fig. 14 that each constant value of the V_1 magnitude produces a specific ellipse as a geometric locus.

By fulfilling simultaneously these two conditions, namely the geometric locus with a constant VUF and a fixed V_1 module, the number of unbalance conditions which produce a constant VUF is reduced. This does not eliminate the possibility of phasors existing within the same ellipse, which generate different effects.

Again, it is apparent that further investigation needs to be conducted to determine the effects of the application of unbalances on electrical equipment, in order to better address quantification problems.

VII. Conclusion

This article presented a detailed investigation of the behavior of the VUF resulting from variations of voltage magnitudes and angles. The identification of problems arising from the use of the VUF to quantify voltage unbalances was one of the fundamental aspects emphasized. Consequently, it was possible to verify that the VUF is more sensitive to variations in voltage angles than to variations in voltage magnitudes. This study also showed that several unbalance conditions may generate a single VUF value. This indicates the need to conduct further research aimed at identifying the geometric loci of the voltage magnitudes and angles for the mentioned specificities.

It was shown in this study that the geometric locus of the magnitudes for identical VUFs is a cylinder which assumes the shape of an ellipse. There are magnitude values inside the cylinder which produce VUFs lower than 2%. Outside the cylinder however, there are VUF values greater than 2%. The geometric locus of the voltage angles for equal VUF values is an ellipse. In this case, it was observed that the geometric loci of the angles for a VUF equal to 2% are identical, under conditions where voltage magnitudes of 201, 220 and 231 volts were considered separately.

Based on the above mentioned geometric loci, a series of voltage phasors was identified which lead to a VUF equal to 1.5%. This VUF value would probably provoke more damaging effects to electric equipment than others where the VUF is equal to 2%, due to the unbalance conditions.

Another important conclusion of this study is the fact that certain voltage magnitude and angle combinations may generate a VUF equal to 0%. These values, when applied to loads such as induction machines, may probably reduce their lifespan.

For the same VUF, there are several voltage phasors that may generate different effects if applied to electrical system equipment.

Furthermore, it was observed that the CVUF angles do not produce useful information for unbalance analyses, due to the fact that the CVUF angle values vary as voltage angles change.

It was also verified that the association of the VUF with the positive component magnitude, in order to quantify unbalances under conditions where the VUF remains constant, reduces the number of phasors capable of generating different effects, but does not eliminate the problem completely.

However, in view of all the conclusions obtained from this study, it can be observed that the VUF has limitations that render its use unadvisable as a single and sufficient parameter for the quantifying of voltage unbalances. In order to confirm this, further studies are suggested aimed at evaluating the effects of voltage unbalances on electrical equipment, and to identify methodologies capable of addressing this problem.

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IX. Biographies

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Marco De Oliveira (M'91-SM'98) was born on December 20th, 1958 in Rio de Janeiro, Brazil. He graduated from the University of Brasilia (Brazil) in 1982. Subsequently, he was awarded his M.Sc. and Ph.D. degrees from the University of Paris (France) in 1989 and 1994 respectively. From 1982 to 1988 he worked at the Operation Division of *Eletronorte* (Brazil) where he was involved with load flow, stability and electromagnetic transient studies. His research interests include power electronics, power quality, renewable sources and energy efficiency. He has published over fifty papers in these fields.

Francisco Assis de Oliveira Nascimento graduated in Electrical Engineering from the University of Brasília, Brazil, in 1982. In 1985, he obtained his M.Sc. degree in Electrical Engineering, and in 1988, a Ph.D. degree, both at the Federal University of Rio de Janeiro, Brazil. Presently, he is an Assistant Professor at the University of Brasília. He has extensive experience in Electrical Engineering, especially in the area of digital signal processing, with emphasis in image codification, data compression, digital communications, fast algorithms and signal processors.